RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2016-19]
B.A./B.Sc. FIRST SEMESTER (July – December) 2016
Mid-Semester Examination, September 2016

Date : 15/09/2016 **MATHEMATICS (General)**
Time : 12 noon – 1 pm Paper : I Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

(*Answer any three questions*) [3×5]

1. State De Moivre's theorem. Use it to prove that $\alpha^n + \beta^n = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$, where α, β are the roots of $x^2 - 2x + 2 = 0$ and n is a positive integer. [2+3]

2. Show that the system of equations
$x + y + z = 4$
$2x - y + 3z = 1$
$3x + 2y - z = 1$
is consistent. Hence solve it. [5]

3. Show that the determinant
$$\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$ is a perfect square. [5]

4. a) Prove that every strictly monotone function is injective. [2]
b) Use the $(\in\text{-}\delta)$ definition of limit to prove that $\lim_{x\to c}\sin x = \sin c$. [3]

5. A function 'f' is defined as
$$f(x) = \begin{cases} 3 + 2x & \text{for } -\frac{3}{2} \le x < 0 \\ 3 - 2x & \text{for } 0 \le x < \frac{3}{2} \\ -3 - 2x & \text{for } x \ge \frac{3}{2} \end{cases}$$
Show that f is continuous at x = 0 and discontinuous at $x = \frac{3}{2}$. [5]

Group – B

(*Answer any four questions*) [4×2·5]

6. Show that $(A \cup B) \cap (A \cup B') = A$ where A, B are subsets of the universal set X and B' is the complement of B.

7. Let $f : A \to B$ be an onto mapping and S, T are subsets of B. Prove that $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$.

8. If $f : A \to B$ and $g : B \to C$ be two mappings such that $g \circ f : A \to C$ is injective then show that f is injective.

9. Give an example of a mapping $f:\mathbb{Z}\to\mathbb{Z}$ ($\mathbb{Z}$ = set of all integers), which is injective but not surjective.

10. Let $S=\{a+b\sqrt{5}:$ a, b are rational numbers$\}$. Show that S – {0} is a commutative group under multiplication.

11. Prove that a group G is abelian if $b^{-1}a^{-1}ba=e$ for all $a,b\in G$ where 'e' is the identity in G.

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